ISSN 2229-5518

Adaptive Fuzzy Model-Based Predictive Control of Nonholonomic Wheeled Mobile Robots Including Actuator Dynamics

Z. Sinaeefar, M. Farrokhi

Abstract— This paper presents an adaptive nonlinear model-based prediction control (NMPC) for trajectory tracking of wheeled mobile robots (WMRs). Robot dynamics are subject to various uncertainties including parameter variations, unknown nonlinearities of the robot and torque disturbances from the environment. In this paper, a discrete-time fuzzy model in combination with NMPC is described to allow approximation of the unknown dynamics of the robot, including the actuator dynamic. Moreover, by tuning the weighting parameters in the cost function of the NMPC, the tracking error of a given trajectory can be minimized. Finally, the parameters of the fuzzy model may be adjusted on-line by the use of a gradient descent algorithm in consideration of the uncertainties. The simulation results of a WMR example show the effectiveness of the proposed method.

Index Terms— Gradient descent algorithm, Fuzzy system, Nonlinear model predictive control, Adaptive control, Mobile robots, Trajectory tracking.

---

1 Introduction

Nowadays, robots are being inserted more and more into dynamic environments such as robotic soccer, manufacturing plants, etc. Trajectory tracking control for mobile robots is a fundamental problem, which has been intensively investigated in the robotics community.

The design of control laws for mobile robots with a dynamic model is considered in several papers, for instance in trajectory tracking [1], [2], [3]. One of the early studies of this problem used a Lyapunov function to design a local asymptotic tracking controller. Global tracking was explored by dynamic feedback linearization techniques in [4], [5], [6], backstepping techniques in [7], [8], [9], and sliding mode techniques in [10]. These controllers require that linear and angular velocities must not converge to zero, so they can not be used for the regulation problem of nonholonomic mobile robots. Also, these controllers do not take into account the restrictions in the control signals due to the difficulty of implementation.

Model predictive control (MPC), also known as receding horizon control (RHC), has become one of the most successful control strategies developed during the last few decades, and unlike many other advanced control techniques, it has desirable features suitable for industrial applications [11], and its applications are also expanding to robot control.

In MPC, a process plant is used to predict future outputs over a prescribed period. Properties that set MPC apart from other control laws are its on-line optimization and constraints. Recent reviews of MPC algorithms and technologies can be found in [12], [13].

However, the possible applications of MPC are limited to linear systems. Where linear models are not sufficiently accurate, the identification of non-linear models for control becomes absolutely necessary. Therefore, the more challenging task of developing a nonlinear MPC (NMPC) has also been attempted [14]. A reactive trajectory tracking controller based on nonlinear model predictive control has been presented in [15]. A nonlinear model predictive control scheme with obstacle avoidance for trajectory tracking of a mobile robot has been proposed in [16]. More examples can be found in [17], [18], [19], [20].

The success of any MPC implementation depends on the effectiveness of the solution method used. One possible and very promising approach to dynamic optimization is to apply intelligent algorithms such as Neural Networks (NN) and fuzzy systems, which have been used in controller designs to deal with various uncertainty problems in the system.

A path tracking scheme for a mobile robot based on fuzzy logic predictive control is presented in [23], where predictive control is used to predict the position and the orientation of the robot, while the fuzzy control is used to deal with the nonlinear characteristics of the system.

The main contribution of this paper is the development of an NMPC for tracking control of WMRs. In the proposed controller, a fuzzy model with parameters adapted on-line is used to estimate the dynamics of the robot. It is assumed that there are uncertainties in both kinematic and dynamic parameters and actuator parameters. To deal with the uncertainties, an adaptive controller is designed using a gradient descent algorithm. The proposed method is applied to a type (2, 0) WMR.

The rest of this paper is organized as follows: In Section 2, the WMR dynamics and the NMPC strategy are presented. Section 3 describes the adaptive NMPC design. Section 4 shows simulation results, and finally, conclusions are given in Section 5.

- *Author name is currently pursuing masters degree program in electric control engineering in Iran University of Science and Technology, Iran. E-mail: sinaeefar@gmail.com*
- *Co-Author name is currently pursuing Associate Professor in electric control engineering in Iran University of Science and Technology, Iran. E-mail: farrokhi@iust.ac.ir*

2 Problem Formulation

2.1 Dynamics of WMR

Using the Euler-Lagrange formulation, the dynamics of WMRs can be described by [24], [25], [26]:

$$\mathbf{M(q)\ddot{q}}+\mathbf{C(q,\dot{q})\dot{q}}+\mathbf{F(\dot{q})}+\mathbf{G(q)}+\boldsymbol{\tau}_d=\mathbf{B(q)}\boldsymbol{\tau}-\mathbf{A}^T\mathbf{(q)}\boldsymbol{\lambda} \quad (1)$$

where $\mathbf{M(q)}\in\Re^{n\times n}$ is a symmetric, positive definite inertia matrix, $\mathbf{C(q,\dot{q})}\in\Re^{n\times n}$ is the centripetal and Coriolis matrix, $\mathbf{F(\dot{q})}\in\Re^{n\times 1}$ is the vector of surface friction, $\mathbf{G(q)}\in\Re^{n\times r}$ is the gravitational vector, $\boldsymbol{\tau}_d$ denotes bounded unknown disturbances including unstructured unmodeled dynamics, $\mathbf{B(q)}\in\Re^{n\times r}$ is the input transformation matrix, $\boldsymbol{\tau}\in\Re^{r\times 1}$ is the input vector, $\mathbf{A(q)}\in\Re^{m\times n}$ is the matrix associated with the constraints, and $\boldsymbol{\lambda}\in\Re^{m\times 1}$ is the vector of constraint forces.

Surface friction is as follows:

$$f(\dot{\mathbf{q}})=F_v\dot{\mathbf{q}}_i+F_d\,\mathrm{sgn}(\dot{\mathbf{q}}_i) \quad (2)$$

where F_v is the coefficient of the viscous friction and F_d is the coefficient of the dynamic friction.

The dynamics of the DC servomotors which drive the wheels of the robot can be expressed as follows:

$$\begin{cases}\boldsymbol{\tau}_s=\mathbf{K}_T\mathbf{i}_a\\ \mathbf{L}\dot{\mathbf{i}}_a+\mathbf{R}\mathbf{i}_a+\mathbf{K}_e\dot{\phi}_e=\mathbf{u}\end{cases} \quad (3)$$

where $\boldsymbol{\tau}_e\in\Re^n$ is the vector of torque generated by the motor, $\mathbf{K}_T\in\Re^{n\times n}$ is the positive definite constant diagonal matrix of the motor torque constant, $\mathbf{i}_a\in\Re^n$ is the vector of armature currents; $\mathbf{L}$, $\mathbf{R}$, and $\mathbf{K}_e$ are the diagonal matrix of armature inductance, armature resistance and back electromotive force constant of the motors, respectively; $\dot{\phi}_e$ is the angular velocities of the actuator motors.

The motor torque $\boldsymbol{\tau}_s$ and the wheel torque $\boldsymbol{\tau}$ are related by gear ratio $\mathbf{N}$ as:

$$\boldsymbol{\tau}=\mathbf{N}\boldsymbol{\tau}_s \quad (4)$$

where $\mathbf{N}$ is a positive definite constant diagonal matrix, and the angular velocities of the actuators $\dot{\phi}_e$ are related to the wheel angular velocities $\mathbf{v}_w$ as:

$$\mathbf{v}_w=\mathbf{N}^{-1}\dot{\phi}_e \quad (5)$$

By ignoring the armature inductance and considering relations (4)-(5), Eq. (3) can be defined as follows:

$$\boldsymbol{\tau}=K_1\mathbf{u}-K_2\mathbf{v}_w \quad (6)$$

where $K_1=\left(\mathbf{NK}_T/\mathbf{R}_a\right)$, $K_2=\mathbf{NK}_eK_1$. The relation between the wheel angular velocities $\mathbf{v}_w$ and the velocity vector $\mathbf{v}$ is:

$$\mathbf{v}_w=\begin{pmatrix}\omega_r\\ \omega_l\end{pmatrix}=\begin{pmatrix}\frac{1}{r} & \frac{b}{r}\\ \frac{1}{r} & -\frac{b}{r}\end{pmatrix}\mathbf{v}\equiv\Sigma\mathbf{v} \quad (7)$$

Substituting (6) and (7) in (1), the equation of WMR, including actuator dynamics, can be obtained as:

$$\mathbf{M(q)\ddot{q}}+\mathbf{C(q,\dot{q})\dot{q}}+\mathbf{F(\dot{q})}+\mathbf{G(q)}+\boldsymbol{\tau}_d=\mathbf{B(q)}\,(K_1\mathbf{u}-K_2\Sigma\mathbf{v})-\mathbf{A}^T\mathbf{(q)}\boldsymbol{\lambda} \quad (8)$$

The kinematic model of WMR can be expressed as follows:

$$\dot{\mathbf{q}}=\mathbf{S(q)v} \quad (9)$$

By taking the time derivative of the kinematic model (8), the robot dynamics (8) can be transformed to:

$$\bar{\mathbf{M}}\dot{\mathbf{v}}+\bar{\mathbf{C}}\mathbf{v}+\bar{\mathbf{F}}+\bar{\boldsymbol{\tau}}_d=K_1\bar{\mathbf{B}}\,\mathbf{u} \quad (10)$$

where

$$\bar{\mathbf{M}}=\mathbf{S^TMS},\quad \bar{\mathbf{C}}=\mathbf{S^TM\dot{S}}+\mathbf{S^TCS}+K_2\bar{\mathbf{B}}\Sigma \quad (11)$$
$$\bar{\mathbf{B}}=\mathbf{S^TB},\ \bar{\mathbf{F}}=\mathbf{S^TF},\ \bar{\boldsymbol{\tau}}_d=\mathbf{S^T}\boldsymbol{\tau}_d$$

According to (11), the wheel actuator input voltages are considering as the control inputs.

2.2 Model Predictive Control Algorithm

The MPC is an optimal control which uses predictions of the system output to calculate the control law [27]. At each sampling instant, the model of the system is used to predict the output of the system over a prediction horizon N_p, and by minimizing a predefined objective function, the future sequence of control inputs are computed.

By use of the receding horizon strategy, only the first control action in the sequence is applied to the system until the next sampling time is reached [18]. The horizons are then moved one sample period towards the future, and optimization is repeated.

Consider the following nonlinear state-space model:

$$x_{t+1}=f(x_t,u_t) \quad (12)$$

where $x_t\in\Re^n$ and $u_t\in\Re^m$ are the system state and control input, respectively. In this paper, it is assumed that function f in (12) is continuous over $\Re^n\times\Re^m$. By defining error vectors $\tilde{x}=x-x_r$ and $\tilde{u}=u-u_r$, we can formulate the cost function as follows:

$$J(k)=\sum_{j=1}^{N_p}\tilde{\mathbf{x}}^T(k+j-1|k)\,\mathbf{Q}\,\tilde{\mathbf{x}}(k+j-1|k)+\sum_{j=1}^{N_c}\tilde{\mathbf{u}}^T(k+j-1)\,\mathbf{R}\,\tilde{\mathbf{u}}(k+j-1) \quad (13)$$

where N_p, N_c are the prediction horizon and control horizon respectively, and $\mathbf{Q}\geq 0$, $\mathbf{R}\geq 0$ are weighting matrices for the error vectors of state and control variables respectively.

We consider also a constraint applied to the amplitude of the control variable:

$$\mathbf{u}_{\min}\leq\mathbf{u}(k+j|k)\leq\mathbf{u}_{\max} \quad (14)$$

Hence, the nonlinear optimization problem can be expressed as follows:

$$\mathbf{u}^*=\arg\min_{\mathbf{x},\mathbf{u}}\{J(k)\} \quad (15)$$

Such that:

$$\mathbf{X}(k|k)=\mathbf{x}_0$$
$$\mathbf{X}(k+j+1|k)=f_d(\mathbf{X}(k+j|k),\mathbf{u}(k+j|k) \quad (16)$$
$$\mathbf{u}_{\min}\leq\mathbf{u}(k+j|K)\leq\mathbf{u}_{\max}$$

At each sampling time k, the optimization problem (15) can be solved, yielding a sequence of optimal control signals $\{\mathbf{u}^*(k|K),\ldots,\mathbf{u}^*(k+N_c-1|K)\}$. Then the first element of the sequence of optimal controls, $\mathbf{u}^*(k|K)$, can be applied to the optimization problem as the actual control action. This procedure repeats at time $k+1$.

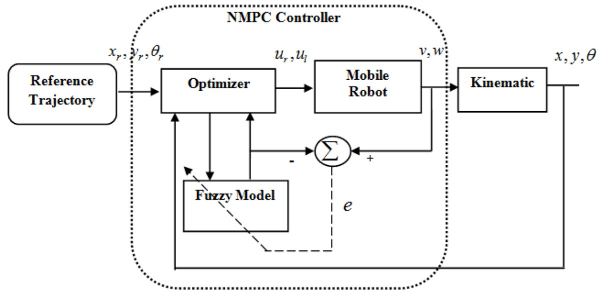


Fig. 1. Block diagram of the proposed NMPC

3 Adaptive NMPC Design

The purpose of trajectory tracking of WMRs is to obtain a control law based on an adaptive fuzzy NMPC technique. The overall control structure is shown in Fig. 1. Fuzzy systems are appropriate candidates for modeling and control of nonlinear systems. An adaptive fuzzy system is defined as a fuzzy logic system whose rules are developed through a training process.

The proposed fuzzy model is used to approximate the model of a mobile robot, including actuator dynamics in order to predict the future output. Also the gradient descent algorithm is employed to adapt the parameter uncertainties.

The governing equation of a mobile robot, including actuator dynamics, can be described generally as a nonlinear discrete system:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)) \tag{17}$$

where $\mathbf{x} = [\mathbf{v}, \dot{\mathbf{v}}]^T$ is the vector of system states and $\mathbf{u}(k) = [u_r(k), u_l(k)]^T$ is the vector of input voltages.

In the proposed control scheme, a fuzzy model is used to estimate the model of a mobile robot with actuator dynamics (17) for predicting the robot behavior. The fuzzy model consists of two parallel fuzzy systems as shown in Fig. 2.

Each fuzzy system has three inputs and one output. The vectors $\left[v(k-1), \omega(k-1), u_r(k-1)+u_l(k-1)\right]^T$, and $\left[\omega(k-1), v(k-1), u_r(k-1)-u_l(k-1)\right]^T$ are the first and second fuzzy system input variables, respectively.

The parameters $u_r(k-1)$ and $u_l(k-1)$ denote the right wheel voltage and left wheel voltage of the robot. The outputs are linear velocity $v(k)$, and angular velocity $\omega(k)$ at time instant k.

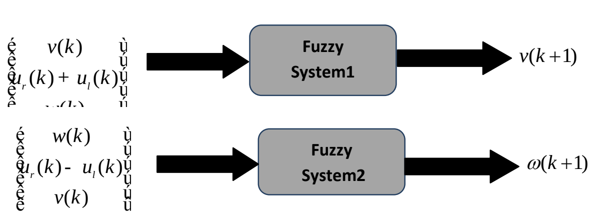


Fig. 2. Block diagram of Fuzzy model

From the robot dynamics (10), (11), $\dot{v}$ and $\dot{\omega}$ can be obtained as:

$$\begin{aligned} \dot{v} &= \frac{K_1}{mr}(u_r + u_l) - \frac{2K_2}{mr^2}v + \frac{(m - 2m_w)\dot{\omega} d}{m}w \\ \dot{w} &= \frac{K_1 b}{rl}(u_r - u_l) - \frac{m_c d}{l}v - \frac{2K_2 b^2}{l\, r^2} \end{aligned} \tag{18}$$

where $l = (I + (m_c - 2m_w)d^2)$.

The inputs $u_r(k-1)+u_l(k-1)$ and $u_r(k-1)-u_l(k-1)$ are selected for fuzzy systems because in (18), the coefficients of u_r and u_l in the equation used to calculate $\dot{v}$ and the coefficients of u_r and $-u_l$ in the equation used to calculate $\dot{\omega}$ are identical.

The membership functions of inputs and outputs are shown in Fig. 3. The next step is the creation of the fuzzy rules based on sample data obtained from the approximate robot dynamics (17). The fuzzy rule-base which is shown in Tables 1 and 2, contains rules covering all combinations of membership functions of the 3 input variables, giving a total of 45 rules.

In this paper we use the set of fuzzy system which includes a singleton fuzzifier, a product inference engine, and the center-average defuzzifier.

The set of fuzzy system can be expressed as follows:

$$f(x) = \frac{\sum_{l=1}^{M} \bar{y}^l \left[\prod_{i=1}^{n} \exp\left(-\frac{\left(x_i - \bar{x}_i^l\right)^2}{2\left(\sigma_i^l\right)^2} \right) \right]}{\sum_{l=1}^{M} \left[\prod_{i=1}^{n} \exp\left(-\frac{\left(x_i - \bar{x}_i^l\right)^2}{2\left(\sigma_i^l\right)^2} \right) \right]} = \frac{a}{b} \tag{19}$$

$$a = \sum_{l=1}^{M} \bar{y}^l z^l, \quad b = \sum_{l=1}^{M} z^l \tag{20}$$

$$z^l = \prod_{i=1}^{n} \exp\left(-\frac{\left(x_i - \bar{x}_i^l\right)^2}{2\left(\sigma_i^l\right)^2} \right) \tag{21}$$

where M and n are constants which denote the number of fuzzy rules and inputs respectively.

The fuzzy membership function that is used in this paper is a Gaussian-shaped form with a centroid $\bar{x}_i^l$ and a width σ_i^l. Also, $\bar{y}^l$ is the centroid of the fuzzy membership function corresponding to the lth rule.

The purpose of adjusting the parameters of the fuzzy model is to minimize the adaptive error which is defined as follows:

$$e(k) = \frac{1}{2}\left(f(x(k)) - y(k)\right)^2 \tag{22}$$

where $f(x)$ is the fuzzy output, and $y(k)$ is the real output of the plant at time k, and $e(k)$ is the error at time k. The parameters of the fuzzy model are updated on-line via the following gradient descent method:

$$\bar{y}^l(k+1) = \bar{y}^l(k) - \alpha \frac{\partial e(k)}{\partial \bar{y}^l(k)}, \quad \frac{\partial e(k)}{\partial \bar{y}^l(k)} = \frac{\left(f(x(k)) - y(k)\right)}{b} \tag{23}$$

ISSN 2229-5518

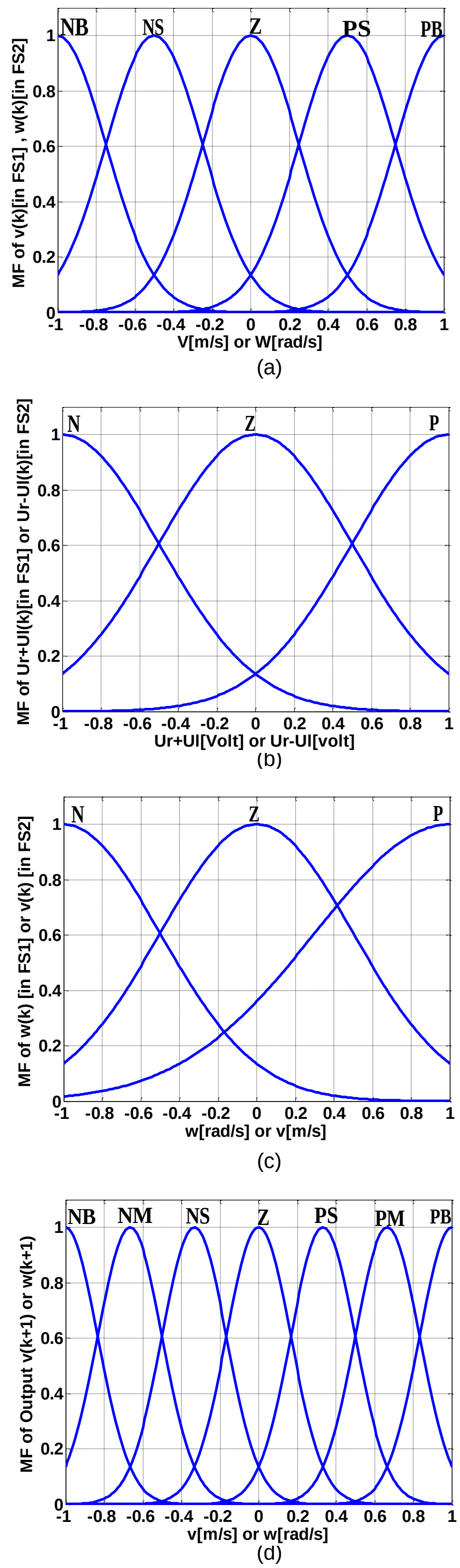


Fig. 3. Membership functions of (a) first input, (b) second input, (c) third inputs, and (d) the output, of each fuzzy system

TABLE 1

FUZZY RULE-BASE OF FUZZY SYSTEM1 FOR THE INPUTS: (A) $u_r(k)+u_l(k)$ Î N, (B) $u_r(k)+u_l(k)$ Î Z, AND (C) $u_r(k)+u_l(k)$ Î P.

$w(k)$ \ $v(k)$	NB	NS	Z	PS	PB
N	NB	NM	NS	Z	PM
Z	NB	NM	Z	Z	PS
P	NB	NM	NM	NB	Z

(A)

$w(k)$ \ $v(k)$	NB	NS	Z	PS	PB
N	NM	NM	Z	NM	PM
Z	NM	Z	Z	PS	PS
P	NB	NM	NM	NM	PS

(B)

$w(k)$ \ $v(k)$	NB	NS	Z	PS	PB
N	NS	PS	PS	PM	PB
Z	NS	Z	Z	PS	PM
P	NB	NM	NM	NM	PM

(C)

$$\bar{x}^l(k+1) = \bar{x}^l(k) - \alpha \frac{\partial e(k)}{\partial \bar{x}_i^l(k)},$$

$$\frac{\partial e(k)}{\partial \bar{x}_i^l(k)} = \frac{([f(x(k)) - y(k)][\bar{y}^l - f(x(k))]z^l[x_i - \bar{x}_i^l])}{b(\sigma_i^l)^2} \tag{24}$$

$$\sigma_i^l(k+1) = \sigma_i^l(k) - \alpha \frac{\partial e(k)}{\partial \sigma_i^l(k)},$$

$$\frac{\partial e(k)}{\partial \sigma_i^l(k)} = \frac{([f(x(k)) - y(k)][\bar{y}^l - f(x(k))]z^l[x_i - \bar{x}_i^l]^2)}{b(\sigma_i^l)^3} \tag{25}$$

where, $0 \le \alpha \le 1$ is the learning rate of fuzzy system.

4 SIMULATION RESULTS

In this section, some computer simulations are performed to evaluate the performance of the proposed controller. In these simulations, the real physical parameters of the WMR and control parameters are summarized in Table 3.

The parameter m_c is the mass of the platform without the driving wheels and the rotors of the DC motors, m_ω denotes the mass of each driving wheel plus the rotor of its motor, I_c denotes the moment of inertia of the platform without the driving wheels and the rotors of the motors and I_m denotes the moment of inertia of each wheel and the motor rotor about a wheel diameter.

TABLE 2
FUZZY RULE-BASE OF FUZZY SYSTEM 2 FOR THE INPUTS: (A) $u_r(k)-\ u_l(k)$ Î N, (B) $u_r(k)-\ u_l(k)$ Î Z, AND (C) $u_r(k)-\ u_l(k)$ Î P

$v(k)$ \ $w(k)$	NB	NS	Z	PS	PB
N	NB	NB	NS	NS	Z
Z	NB	NS	Z	NM	Z
P	NB	NM	Z	PM	PM

(A)

$v(k)$ \ $w(k)$	NB	NS	Z	PS	PB
N	NB	NB	Z	Z	Z
Z	NM	Z	Z	Z	PM
P	NB	NM	PS	PB	PB

(B)

$v(k)$ \ $w(k)$	NB	NS	Z	PS	PB
N	NM	NM	PS	PS	PM
Z	NM	PS	PS	PS	PM
P	NB	NM	PS	PB	PB

(C)

The kinematic and dynamic matrices in (11) are expressed as:

$$\mathbf{S}(q)=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix},\quad \bar{\mathbf{M}}=\begin{bmatrix} m & 0\\ 0 & I\end{bmatrix}$$

$$\bar{\mathbf{C}}=\begin{bmatrix}\dfrac{2K_2}{r^2} & m_c d\dot{\theta}\\ -m_c d\dot{\theta} & \dfrac{2b^2K_2}{r^2}\end{bmatrix} \quad (26)$$

where $m=m_c+2m_\omega$, $I=I_c+2I_m+m_cd^2+2m_\omega b^2$.

The motor voltage bounds are listed as [-12 v, 12 v]. The controller parameters are selected as $N_p=5, N_c=1$. The sampling time is 0.1 sec. The weighting matrices are assumed as $\mathbf{Q}=diag\{50,30,1\}$, $\mathbf{R}=diag\{0.005,0.005\}$.

As discussed before, the model of the mobile robot with motor dynamics is estimated by fuzzy systems during the online optimization. The learning rate of each fuzzy system uses $\alpha=0.92$.

In order to show the performance of the proposed controller, the fuzzy NMPC was applied to the robot for two cases:

In the first simulation, the adaptive tracking controller is tested only for uncertain parameters. It is assumed that there is no knowledge about the WMR parameters, and there is no disturbance in this case.

The desired trajectory for this case is a circular path which is chosen as follows:

TABLE 3
WMR PARAMETERS

Parameter	Simulation value
r(m)	0.15
b (m)	0.75
d(m)	0.3
L(m)	0.1
m_ω (m)	1
m_c (m)	36
I_m (Kg.m²)	0.0025
I_c (Kg.m²)	15.625
I_ω (Kg.m²)	0.005
T(s)	0.02
K_1	7.2
K_2	2.592

$$x_r(t)=10+7.5\cos(\omega_r t),\\ y_r(t)=25+7.5\sin(\omega_r t) \quad (27)$$

where $\omega_r(t)=0.2$, and $v_r(t)=1.5$.

The initial position of the WMR is selected as $\mathbf{q_0}(t)=[19,24,pi/2]^T$.

Simulation results of the proposed fuzzy NMPC are shown in figures 4 to 6. As these figures show, the WMR can follow the desired path and at the desired velocity with good accuracy. Moreover, the motor voltages are within the predefined bounds. Mean square position and velocity errors for non-adaptive Fuzzy NMPC and adaptive Fuzzy NMPC after reaching the trajectory are given in Table. 4. As shown in the table, the control behavior of the adaptive Fuzzy model base predictive controller is seen to be relatively ideal for tracking a circular path.

In the second case, an external disturbance $\boldsymbol{\tau}_d$ is applied to the WMR at $t=5$ sec. The desired trajectory for this case is a circle and the other control parameters are the same as in case one. The simulation results of the non-adaptive Fuzzy NMPC and the adaptive Fuzzy NMPC are shown in Fig. 7 respectively. As shown in this figure, the non-adaptive Fuzzy NMPC follows the reference trajectory with a sizeable error vesus the adaptive Fuzzy NMPC.

As these figures show, the adaptation capability of the fuzzy system can cope with this disturbannce very quickly and return the mobile robot to its desired path, yielding an adaptive and robust control method.

TABLE 3
MEAN SQUARE POSITION AND VELOCITY ERRORS FOR NON-ADAPTIVE FUZZY NMPC AND ADAPTIVE FUZZY NMPC

		MSE
Mode Non-Adaptive	Position	0.3445
	Velocity	0.0453
Mode Adaptive	Position	0.099
	Velocity	0.0019

ISSN 2229-5518

5 Conclusion

To achieve better path tracking for WMRs, an adaptive fuzzy NMPC control method was designed in this paper. The proposed controller solves the integrated kinematic and dynamic tracking problem in the presence of both parametric and non-parametric uncertainties. Furthermore, a fuzzy system whose parameters are updated on-line by a gradient descent algorithm has been employed. While this fuzzy system can provide an appropriate model of the robot, it can also deal with any changes in robot parameters.

The simulation results on a type (2, 0) WMR illustrate the effectiveness of the proposed control scheme. Future work should focus on the stability analysis of the proposed method.

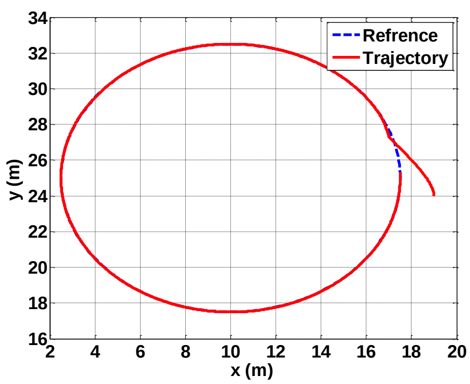
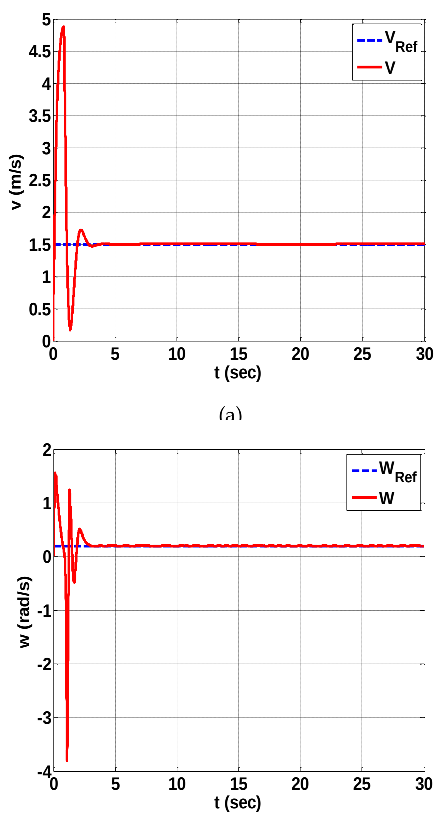
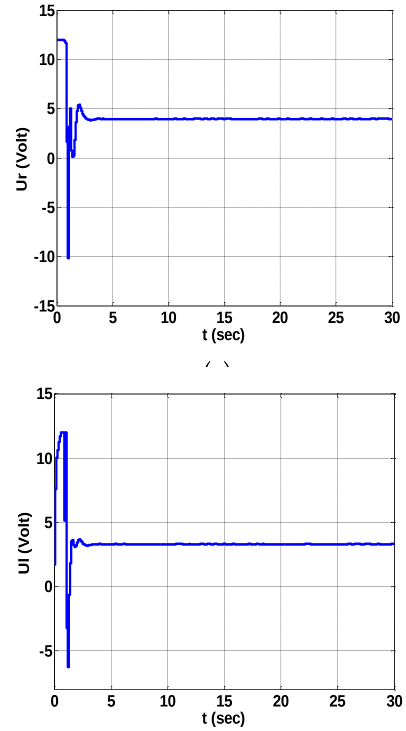


Fig. 4. Desired and actual trajectories for WMR



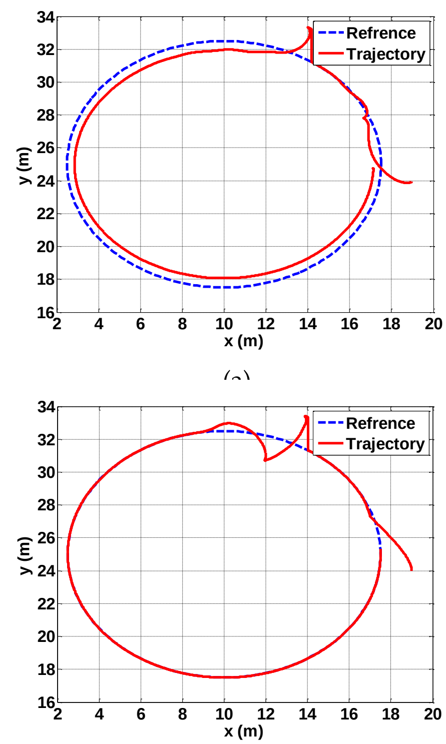
(b)

Fig. 5. Desired and actual velocities of WMR



(b)

Fig. 6. Control voltages of the WMR for: (a) the right wheel, (b) the left wheel



(b)

Fig. 7. Desired and actual trajectories for WMR in presence of external disturbances for: (a) Non-Adaptive Fuzzy NMPC, and (b) Adaptive Fuzzy NMPC

REFERENCES

[1] T. Das and I. N. Kar, "Design and Implementation of an Adaptive Fuzzy Logic-Based Controller for Wheeled Mobile Robots," *IEEE Transaction on Control Systems Technology* , vol. 14, no. 3, pp. 501-510, 2006.

[2] C. Chen, T. S. Li and Y. C. Yeh, "EP-Based Kinematic Control and Adaptive Fuzzy Sliding-Mode Dynamic Control for Wheeled Mobile Robots," *Information Sciences,* vol. 179, no. 1-2, pp. 180-195, 2009.

[3] F. Abdessemed, KH.Benmahammed, and E.Monacelli, "A Fuzzy-Based Reactive Controller for a Non-Holonomic Mobile Robot," *Robotics and Autonomous Systems*, vol. 47, pp. 31–46, 2004.

[4] A. De Luca and M. D. Di Benedetto, "Control of Nonholonomic Systems via Dynamic Compensation," *Kybernetica*, vol. 29, no. 6, pp.593-608, 1993.

[5] B. d'Andrea-Novel, G. Campion, and G. Bastin, "Control of Nonholonomic Wheeled Mobile Robots by State Feedback Linearization," *Int. J. Robot. Res.*, vol. 14, no. 6, pp. 543-559, 1995.

[6] C. Samson and K. Ait-Abderrahim, "Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space," *Proceedings of IEEE International Conference on Robotics and Automation,* pp. 1136-1141, 1991.

[7] R. Fierro and F. L. Lewis, "Control of a Nonholonomic Mobile Robot: Backstepping Kinematics into Dynamics," *Proceedings of 34th IEEE CDC,* pp. 3805-p3810, New Orleans, 1995.

[8] G. Indiveri, "Kinematic Time-Invariant Control of a 2-D Nonholonomic Vehicle," *Proceedings of 38th IEEE CDC,* pp. 2112-2117, 1999.

[9] Z. P. Jinag and H. Nijmeijer, "Tracking Control of Mobile Robots: A Case Study in Backstepping," *Automatica,* vol. 33, pp. 1393-1399, 1997.

[10] A. M. Bloch and S. Drakunov, "Tracking in Nonholonomic Dynamic Systems via Sliding Modes," *Proceedings of 34th IEEE CDC,* pp. 2103-2106, New Orleans, 1995.

[11] E. F. Camacho, and C. Bordons, "*Model Predictive Control in the Process Industry,"* Springer, Verlage, New York, 1995.

[12] S. J. Qin and Thomas A. Badgwell, "A Survey of Industrial Model Predictive Control Technology," *Control Engineering Prectice*, vol. 11, pp. 733-764, 2003.

[13] E. F. Camacho and C. Bordons, *Model Predictive Control,* Springer, London, 2004.

[14] R.Findeisen, and F.Allgower, "*An Introduction to Nonlinear Model Predictive Control,"* Technical Report, University of Stuttgart, Germany, 2002.

[15] S. G. Vougioukas, "Reactive Trajectory Tracking for Mobile Robots Based on Non Linear Model Predictive Control," *Proceedings of IEEE International Conference on Robotics and Automation*, pp. 10-14, Italy, 2007.

[16] H. Lim, Y. Kang, CH. Kim, J. Kim, and B. You, "Nonlinear Model Predictive Controller Design with Obstacle Avoidance for a Mobile Robot," *Proceedings of IEEE International Conference on Mechtronic and Embedded Systems and Applications,* pp. 494-499, Beijing, 2008.

[17] A. S. Conceicao, H. P. Oliveira, A. S. Silva, D. Oliveira, and A.P.Moreira, "A Nonlinear Model Predictive Control of an Omni-Directional Mobile Robot," *Proceedings of IEEE International Symposium on Industrial Electronics*, pp. 2161-2166, 2007.

[18] A. S. Conceicao, A. P. Moreira, and P. J. Costa, "A Nonlinear Model Predictive Control Strategy for Trajectory Tracking of a Four-Wheeled Omnidirectional Mobile Robot," *Optimal Control Applications and Methods*, vol. 29, no. 5, pp. 335-352, 2008.

[19] K. Kanjanawanishkul and A. Zell, "Path Following for an Omnidirectional Mobile robot Based on Model Predictive Control," *Proceedings of IEEE International Conference on Robotics and Automation*, pp. 257-262, USA, 2009.

[20] K. Kanjanawanishkul, M. Hofmeister, and A. Zell, "Smooth Reference Tracking of a Mobile Robot Using Nonlinear Model Predictive Control," *Proceedings of the 4th European Conference on Mobile Robots (ECMR)*, pp. 161-166, Croatia, 2009.

[21] Gu. Dongbing and Hu .Huosheng, "Neural Predictive Control for a Car-Like Mobile Robot," *Robotics and Autonomous Systems*, vol.39, no. 2-3, pp.73-86, 2002.

[22] J. Gomez-Ortega and E. F. Camacho, "Neural Network MBPC for Mobile Robots Path Tracking," *Robotics and Computer Integrated Manufacturing*, vol. 11, no. 4, pp. 271-278, 1994.

[23] J. Xianhua, M. Yuichi, and Z. Xingquan, "Predictive Fuzzy Control for a Mobile Robot with Nonholonomic Constraints," *Proceedings of IEEE International Conference on Advanced Robotics*, pp. 58-63 , Seatle WA , 2005.

[24] R. Fierro and F. L. Lewis, "Control of a Nonholonomic Mobile Robot Using Neural Networks," *IEEE Transaction on Neural Netorks*, vol. 9, no. 4, pp. 589–600, 1998.

[25] R. Fierro and F. L. Lewis, "Control of a Nonholonomic Mobile Robot: Backstepping Kinematics into Dynamics," *Proceedings of IEEE International Conference on Decision and Control*, vol. 4, pp. 3805-3810, LA, 1995.

[26] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive Tracking Control of a Nonholonomic Mobile Robot," *IEEE Transaction on Robotics and Automation*, vol. 16, no. 5, pp. 609–615, 2000.

[27] F. Allgower, R. Findeisen, Z. K. Nagy, "Nonlinear Model Predictive Control: From Theory to Application," *Chinese Institute of Chemical Engineers*, vol. 35, no. 3, pp. 299-315, 2004.

[28] R. Fierro and F. L. Lewis, "Control of a Nonholonomic Mobile Robot Using Neural Networks," *IEEE Transaction on Neural Netorks*, vol. 9, no. 4, pp. 589–600, 1998.